AN ELEMENTARY COURSE IN

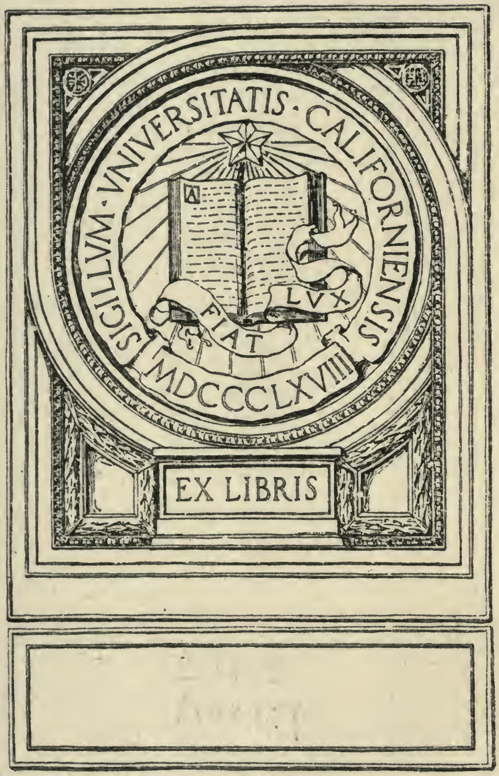
GRAPHIC MATHEMATICS

BY

MATILDA AUERBACH

INSTRUCTOR IN MATHEMATICS, ETHICAL CULTURE HIGH SCHOOL
NEW YORK CITY

ALLYN AND BACON
Boston New York Chicago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PREFACE

The object of this little book is threefold:—first, to show the pupil some practical uses of the graphic method; second, to plan a course in graphic algebra that will lead naturally and along interesting paths to the work in the solution of equations; and finally to save both teacher and pupil time and energy needed to hunt up suitable material.

Every type of work outlined in the book has been tested and found suitable for classroom use. The writer has done a considerable amount of work in this line with her classes for the past nine years, and has never failed to find it a spring by means of which she has been enabled to arouse an interest in the mathematics.

Though elementary in its form, it is believed the monograph will be found to be thoroughly scientific. It endeavors to introduce in simple form ideas which the pupil will come to deal with in more advanced work and in no case introduces an idea which must sooner or later be unlearned.

In the Appendix at the end of the book may be found a number of statistical tables, obtained chiefly from the Bureau of Statistics at Washington, from which teacher and pupil may freely draw without waste of time. The writer has aimed to cover a wide variety of topics and at the same time to select those in which figures were not too large for convenient use.

MATILDA AUERBACH,
ETHICAL CULTURE HIGH SCHOOL.

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CHAPTER I

INTRODUCTORY: THE MEANING OF A GRAPH

We all have had the experience of wishing to place a point somewhere definitely upon a sheet of paper, upon the blackboard, or upon some flat surface. How have we done it? What have we really done when we have said the point is to be three inches from the lower edge and two inches from the right edge? We have done practically what we do when we say New York City is 74° West longitude and 41° North latitude. We have drawn two lines (either real or imaginary) in the first case, one three inches above the lower edge and the other two inches to the left of the right edge of the paper, and have found the point at their crossing—in the second case we have drawn one line through a point on the equator just 74° to the left of the meridian through Greenwich, and another line parallel to the equator just 41° above it. Their point of intersection has again given us the desired point. In the same manner we could construct any map—one of the city, showing points of interest—one of a piece of ground that has been surveyed, or anything of the sort, just by referring each of the points in question to two intersecting lines. These lines are known as axes, and in all elementary work are drawn at right angles to each other.

EXERCISES

1. If West longitude is reckoned to the left of the Greenwich axis, how will East longitude be reckoned? If North latitude is reckoned up from the equator, how will South latitude be reckoned?

2. Using the Greenwich meridian and equator as axes, locate the following cities:

(1) New York (74° W., 41° N.)
(2) St. Petersburg (30° E., 60° N.)
(3) Buenos Ayres (58° W., 35° S.)
(4) San Francisco (122° W., 37° N.)
(5) Zanzibar (49° E., 6° S.)
(6) London (0°, $51\frac{1}{2}$° N.)

3. Using any two streets that run at right angles to each other as axes, locate at least a dozen points of interest in the city in which you live.

In locating points in general with respect to two axes, matters may be greatly simplified by using positive and negative numbers.

EXERCISES

1. List the following words and phrases under the two heads "positive" and "negative":—right, wrong; debit, credit; right, left; below, above; above zero, below zero; B. C., A. D.; East, West, North, South; sane, insane; pauper, tax-payer; time to come, time past; increase in population, decrease in population.

2. Which of the above might be considered as lying to the right of a vertical axis? Which to the left? Which above a horizontal axis? Which below it?

We have seen that to locate a point on a plane surface, reference must be made to two axes, for there are innumerable points that lie four inches to the right of a vertical axis, while there is but one that lies at the same time 5 inches below a horizontal axis.

EXERCISES

Suppose we take the turning point from the year 1907 to the year 1908 as our zero point on the horizontal axis in this diagram, (Fig. 1), and the temperature 0° Fahren-

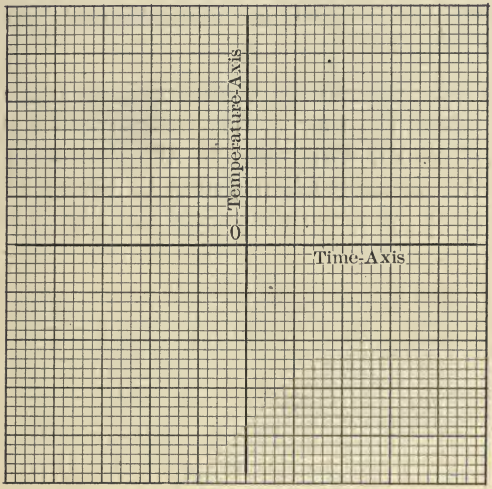


Fig.1

heit as our zero point on the vertical axis:—

1. Where will all points representing time previous to Jan. 1, 1908, be located? Where all those representing time after that date? Where all those representing temperature below zero? and where all those representing temperature above zero?

2. Through what point would you draw an imaginary line to represent mid-day, Jan. 5, 1908, if each day of

24 hours is represented by 12 small divisions on the diagram? Dec. 25, 1907, 6 P. M.? Jan. 10, 1908, 8 A. M.?

3. Through what point would you draw a line to represent the temperature 5° above zero (that is, +5°)? 7° below zero? 12° below zero?

4. Look up the temperature for each day of the past week, and record it by means of a diagram.

For more complicated problems of this type see Appendix to Chapter II.

As you may already have observed, we can in general locate points in the four quadrants into which our surface is divided by the two axes in the following manner. Suppose the distance of all points to the right or left of the vertical or yy' axis in the diagram (Fig. 2) be denoted

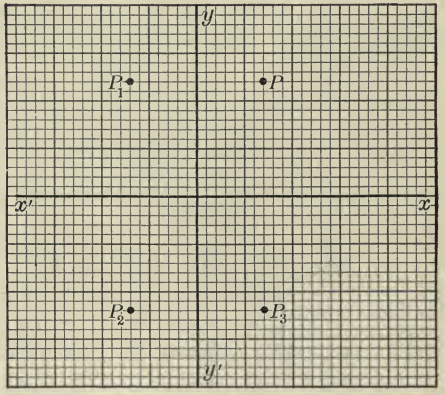


Fig.2

by x, and the distance of all those above or below the the horizontal or xx' axis be denoted by y. Then when x is positive the distance is measured so many units to the right, and when it is negative, so many units to the left of the yy' axis. When y is positive the distance is measured so many units above the xx' axis, and when negative so many below it. For instance, suppose the the point $(x, y) = (7, 12)$ be given. It will be in the first quadrant, (1, Fig. 2), on an imaginary line 7 units to the right of yy' and parallel to it, and on another such line 12 units above xx' and parallel to it—namely point P. If a point is described as $(x, y) = (-7, 12)$ it will lie in quadrant II, 7 units across to the left, and 12 units up, namely point P_1. $(x, y) = (-7, -12)$ will lie in the third quadrant 7 units across to the left, and 12 down, point P_2, and finally the point $(x, y) = (7, -12)$ lies in quadrant IV, 7 units across to the right, and 12 units down, point P_3.

EXERCISES

1. Locate the points (9, 11), (7, 6), (—15, 17), (—19, —20), (—2, 6), (8, —15), (7, —13), (—11, —9), (—2, 15).

2. Locate the points (1, 5), (3, 7), (5, 2), (9, —3), (12, —6) and draw a line connecting them.

Any line (curved, broken or straight) drawn through a series of fixed points as in the last exercise is called a graph.

EXERCISE

1. Draw the graph determined by the points (—3, —2), (—1, 0), (0, 1), (2, $\frac{1}{2}$), (5, 7), (8, —11).

CHAPTER II

SOME OF THE PRACTICAL USES OF THE GRAPH

Now that we have learned to locate points in this simple manner, we are ready for a few simple practical applications in addition to the above.

IN SURVEYING

EXERCISES

1. In surveying a hexagonal field a surveyor notes the following points as its vertices: $A = (6, 7)$, $B = (20, 20)$, $C = (40, 20)$, $D = (35, 0)$, $E = (10, -20)$ and $F = (0, -10)$. Plot the points, and draw the outline of the field. Find the number of square units in the area of the field in two ways:— (1) By breaking the diagram of the field into figures of which you can find the areas and adding them, (2) By a process of subtraction, using the square whose vertices are denoted by the points (0, 20), (40, 20), (40, —20), (0, —20).

2. It is customary among surveyors to have the polygon lie eventually entirely in the first quadrant. Can you see any reason for this?

3. Through how many units will you have to move the polygon indicated in Ex. 1, so that it shall just lie wholly in the first quadrant?

4. Will all the values indicating the vertices be changed?

5. Describe the new positions of A, B, C, D, E, and F.

6. The vertices of a pentagonal field are located by the following points, $A = (-20, 15)$, $B = (10, 20)$, $C = (23, -20)$, $D = (-10, -30)$, $E = (-30, -10)$.

(1) Draw the outline of the field.

(2) Give new values to A, B, C, D, E, so that the area shall remain the same but the diagram lie wholly in the first quadrant with E on the North-South axis, and D on the East-West axis.

(3) Find the area of the field.

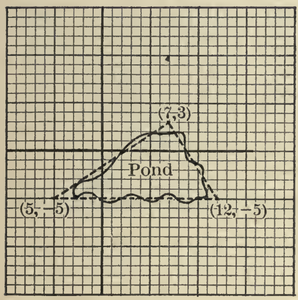


Fig.3

7. From the accompanying diagram (Fig. 3), find the approximate area of the pond.

8. The accompanying diagram (Fig. 4), represents the survey of a field with curved boundary. Find the approximate area of the field.

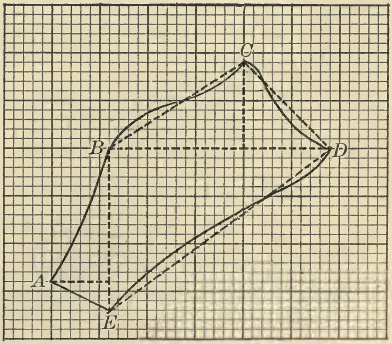


Fig.4

IN KEEPING STATISTICS AND AS READY RECKONERS

9. The following table gives the highest and lowest prices in New York, for Middling Uplands Cotton from Jan. 1 to Dec. 31 of the years named. Show the graph of the highest in red ink and that of the lowest in black ink on the same pair of axes, and correct to the nearest half.

YEAR	HIGHEST	LOWEST	YEAR	HIGHEST	LOWEST	YEAR	HIGHEST	LOWEST
1826	14	9	1864	190	72	1872	$27\frac{3}{8}$	$18\frac{5}{8}$
1835	25	15	1865	120	35	1873	$21\frac{3}{8}$	$13\frac{5}{8}$
1840	10	8	1866	52	32	1874	$18\frac{7}{8}$	$14\frac{3}{4}$
1850	14	11	1867	36	$15\frac{1}{2}$	1885	$13\frac{1}{4}$	$10\frac{15}{16}$
1860	$11\frac{5}{8}$	10	1868	33	16	1890	$12\frac{3}{4}$	$9\frac{3}{16}$
1861	38	$11\frac{1}{2}$	1869	35	25	1895	$9\frac{3}{8}$	$5\frac{9}{16}$
1862	$69\frac{1}{2}$	20	1870	$25\frac{3}{4}$	15			
1863	93	51	1871	$21\frac{1}{4}$	$14\frac{3}{4}$			

10. What facts does the graph of the table in Ex. 9 bring out clearly before you?

11. Calling one the time axis, and the other the population axis, draw graphs indicating the following sets of data:

(1) The population of the United States per square mile:

YEAR	POP.	YEAR	POP.
1800	6.41	1900	25.22
1850	7.78	1904	27.02
1870	12.74		

(2) The population of England, Ireland, Scotland, and Wales correct to the nearest 10,000: (Draw the graphs using a single pair of axes, a different kind of line for each, and correct to the nearest 100,000.)

YEAR	ENGLAND	IRELAND	SCOTLAND	WALES
1831	13,090,000	7,770,000	2,360,000	810,000
1841	15,000,000	8,200,000	2,620,000	910,000
1851	16,920,000	6,570,000	2,890,000	1,010,000
1861	18,950,000	5,800,000	3,060,000	1,110,000
1871	21,500,000	5,410,000	3,360,000	1,220,000
1881	24,610,000	5,180,000	3,740,000	1,360,000
1891	27,500,000	4,710,000	4,030,000	1,500,000
1901	32,530,000	4,460,000	4,470,000	*

* After 1891 merged into England.

12. Answer the following questions from the graphs drawn in Ex. 11, (2):

(1) In approximately what year was the population of England 17 million?

(2) What was the population of England in 1835? in 1845? in 1865? in 1875?

(3) In which of the four countries has the population increased least rapidly? Most rapidly?

(4) In which has there been a decrease?

(5) In what year was the population of two of them practically the same? In which countries was this the case?

(6) Roughly speaking, when will the population of England be 38 million? (*i. e.* considering the increase to continue uniformly.)

(7) What will be the population of each of the others at that time?

(8) When will that of Ireland and Wales be the same? What will it be at that time?

(9) Will this happen apparently in the case of Scotland and Wales?

For other problems of this type see Appendix to Chapter II.

The graphic method of recording the readings of a thermometer and barometer has been adopted by many newspapers.

EXERCISES

1. Observe the readings of the same thermometer at the same hours daily for a week, and record the results of your observations graphically.

2. Record graphically the readings of the barometer as taken from the same newspaper daily for a week.

3. Record graphically the scores of the captains of the girls' and boys' basket ball teams in your school. (One in red and the other in black ink, or one by means of a solid and the other by means of a dotted line.)

4. The Harvard Eights from 1852 through 1905 had rowed 39 races. The records are as follows:

DATE	WON BY	TIME WINNER	TIME LOSER	DATE	WON BY	TIME WINNER	TIME LOSER
1852	Harvard	——	——	1884	Yale	20.31	20.46
1855	"	——	——	1885	Harvard	25.15	26.30
1857	"	19.18	20.18	1886	Yale	20.41	21.05
1859	Yale	19.14	19.16	1887	"	22.56	23.11
1860	Harvard	18.53	19.05	1888	"	20.10	21.24
1864	"	19.01	19.43	1889	"	21.30	21.55
1865	Yale	17.42	18.09	1890	"	21.29	21.40
1866	Harvard	18.43	19.10	1891	Harvard	21.23	21.57
1867	"	18.13	19.25	1892	Yale	20.48	21.42
1868	"	17.48	18.30	1893	"	25.01	25.15
1869	"	18.02	18.11	1894	"	22.47	24.40
1870	"	Foul	Disq.	1895	"	21.30	22.05
1876	Yale	22.02	22.33	1899	Harvard	20.52	21.13
1877	Harvard	24.36	24.44	1900	Yale	21.13	21.37
1878	"	20.45	21.29	1901	"	23.37	23.45
1879	"	22.15	23.58	1902	"	20.20	20.33
1880	Yale	24.27	25.09	1903	"	20.20	20.30
1881	"	22.13	22.19	1904	"	21.40	22.10
1882	Harvard	20.47	20.50	1905	"	22.33	22.36
1883	"	24.26	25.59				

Show this graphically.

As seen above in plotting population curves, valuable surmises might be made in regard to probable increase or decrease in populations during specified periods, or rough estimates could be made as to the probable populations at any stated time, and so forth. Likewise, there is another use of the graph in the way of a "ready-reckoner" where price lists do not include, for instance, all sizes of articles or numbers of articles of the same kind for sale. This will be made clear by the following set of problems:

1. The single ticket by railway costs $2.50. If 10 such tickets be purchased the average cost will be reduced to $2.25. If 50 be purchased the cost per ticket will be only $1.80; if 100, the cost per ticket will be $1.50; and if 200, the cost per ticket will be $1.25. Draw a graph showing this, and answer the following questions by the aid of it:

> (1) What will be the probable cost per ticket if an excursion of 75 be formed? If one of 125 be formed? One of 175?
>
> (2) About how many tickets must be used to reduce the expense per head to just $2.00? to $1.60?

2. If a certain kind of desk be sold to the individual it will cost $30.00. If ordered by the dozen it will cost $28.50, if 6 dozen are ordered it will cost $22.50, and if 150 are ordered the cost will fall as low as $20.00. Draw a graph showing this, and answer the following questions:

> (1) What will be the probable cost per desk when 36 are ordered? When 100 are ordered?
>
> (2) How many must be ordered so that each shall cost about $25.00?

3. Ordering ink by the gill it costs $ .10. By the pint it costs $ .30, by the quart $ .50, and by the gallon

$1.75. According to this, what should it cost approximately when ordered by the half-gallon? By the half-pint? By the quart and a pint?

4. The average annual premiums (P) for whole life insurance of $500 for the age ($A$) at entry is given as follows:

$A =$	21	25	30	35	40	45	50
$P =$	$8.00	$8.66	$10.00	$11.66	$14.00	$16.75	$20.10

What are the probable premiums for ages 23, 27, 33, 37, 42, 48?

5. It is found by testing, that the barometer stands at 30 inches at sea level, at 23.5 inches at a height of 6,000 feet, at 18.2 inches at a height of 12,000 feet, at 12.2 inches at 24,000 feet, and at 7.3 inches at 36,000 feet above sea level. Plot the graph indicating these facts, and from it answer the following questions:—

(1) How high (approximately) is a place in which the barometer stands at 25 inches? At 20 inches?

(2) How high should the barometer rise in a spot which is 20,000 feet above sea level? At one which is 30,000 feet above sea level?

6. In a price list the following table appears:

Measuring-tins of capacity P (pints) =	1	2	3	4	6	8	12
Cost in cents C =	10	16	21	24	30	35	42

What will tins of a capacity of 5 pints, 7 pints, 9, 10, 11 pints respectively, probably cost?

7. The cost of fitted lunch baskets is given in the following table:

Arranged for number of persons $N =$	1	2	4	6
Cost in dollars $D =$	10	18	30	40

What will be the probable cost of baskets for 3, 5, 7, 8, and 10 persons respectively?

IN REPRESENTING FORMULAS

In the last set of applications of the graph we have seen that by joining successive given points by straight lines, we may surmise approximate results for intermediate points. However, there has been no law governing the statements thus made, and the results obtained may or may not satisfy existing conditions. In short, it was only a surmise on our part when we drew conclusions.

There is, however, another type of problem which may be represented or approximately solved graphically—namely those which rest upon a formula. For instance, we are told that the circumference of a circular is always equal to π times its diameter, or approximately $3\frac{1}{7}$ times its diameter. That is, if C stands for the number of units in a circumference, and D for the number of units in its diameter, $C \equiv \pi D$.

EXERCISES

1. Given $C \equiv \frac{22}{7} D$, where $C =$ number units in the circumference of a circle and $D =$ number units in its diameter:—

(1) Find the values of C for those given in the following table for D.

$D =$	7	14	$3\frac{1}{2}$	21	28
$C =$					

(2) Call one axis (DD'), the diameter axis, and the other (CC'), the axis of circumferences, and plot the points corresponding to the values found in Ex. (1).

(3) Connect these points and state on what kind of line they lie.

(4) How many of these points would have been needed to enable you to draw that line?

(5) From the line you have drawn find answers to the following questions:

(*a*) When the diameter of a circle is 10 units how many units are contained in its circumference?

(*b*) When $D = 10\frac{1}{2}$ ft., $C =$?

(*c*) When $C = 100$, $D =$?

(*d*) When $C = 75$, $D =$?

(*e*) If the circumference of a wheel is 92 inches, what is the length of its diameter?

2. We are told that an inch contains 2.54 centimeters. Answer the following:

(1) The number of centimeters in a given length is then always how many times the number of inches in that length?

(2) Write a formula stating this fact.

(3) As in Ex. 1 (1), select any six lengths in terms of inches and make a table showing the

number of centimeters in the corresponding lengths.

(4) Call one axis (II'), and the other (CC'), and plot the points corresponding to the values found in (3).

(5) On what kind of line do these points lie? Draw it.

(6) How many of these points would have been needed to enable you to draw that line?

(7) From the graph just plotted, answer the following questions:

(*a*) About how many inches in 30 cm.?

(*b*) About how many centimeters in 20 in.?

(*c*) About how many inches in 40 cm.?

(*d*) About how many inches in a meter?

3. The formula for the reduction of Fahrenheit scale to Centigrade scale is $C = \frac{5}{9}(F - 32)$ where C = the number of degrees Centigrade corresponding to F = any given number of degrees Fahrenheit.

(1) Give six values to F, and as in Ex. 1 (1), show in a table the corresponding values of C.

(2) Call the axes of Fahrenheit and Centigrade FF' and CC' respectively, and plot the points shown in this table.

(3) Connect these points and tell on what kind of line they lie.

(4) How many of these points would have been needed to enable you to draw that line?

(5) From the lines you have drawn find the approximate number of degrees on a Fahrenheit thermometer when a Centigrade thermometer registers (*a*), 10°, (*b*), 100°, (*c*), 50°, (*d*), 120°, (*e*), 0°.

(6) From the same line find the approximate number of degrees on a Centigrade thermometer when a Fahrenheit thermometer registers (*a*), 10°, (*b*), 20°, (*c*), 35°, (*d*), 180°, (*e*), 212°.

4. On an examination paper 125 points may be obtained.

(1) Write a formula stating this fact and draw its graph as in the above exercises so that the examiner may use it to mark the set of papers. (That is, so that he may reduce any number of points to per cent.)

(2) What per cent. will pupils have who have 90 points, 10 points, 60 points, 115 points, 120 points correct?

GENERAL QUESTIONS

1. In each case in the above four exercises, the formula was of what degree?

2. In each case what was the result in plotting the graph of the formula?

3. In each case how many points were needed to plot the graph of the formula?

4. Can you formulate a general rule as to advisability in the selection of these points?

It has been possible to represent each of the foregoing formulas by means of a straight line. There are, however, many that cannot be so represented. The following problems will make this point clear.

EXERCISES

1. The area of a circle in terms of its radius is expressed by the formula $A = \pi R^2$. Find the values of A when $R = 1, 1\frac{1}{2}$ $2\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}, 7$, and plot the corresponding points. (Let $\pi = 3\frac{1}{7}$, call the axes AA' and RR', and use

a convenient scale.) Will the line drawn through these points be a straight line? Could it have been found from any two of the points used? What would you have to do to find a more accurate graph than the one you have found?

2. From the graph drawn in Ex. 1, answer the following questions:—

(1) What is the approximate area of the circle whose radius is 3, 4, 5 feet respectively?

(2) What is the approximate length of the radius of a circle when its area is 150, 38 square units respectively?

3. When a body falls freely from rest, the space in feet, s, through which it travels in a given time in seconds, t, is expressed by the formula $s = 16\,t^2$. What will be a good scale to use in plotting the graph of this formula? Find the corresponding values of s when

$t =$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{3}{2}$
$s =$							

Plot the graph of the points thus found, using the scale decided upon.

4. From the graph drawn for Ex. 3, what is the approximate distance through which a body falls in 5, $2\frac{3}{4}$, $3\frac{1}{4}$, seconds respectively?

5. From the same graph find the approximate time needed for a body to fall 64 ft., 144 ft., 120 ft.

6. About how high is a building if a ball dropped from the roof takes 3 seconds to reach the ground?

7. If squares of brass are cut from a sheet of uniform thickness, their weights are proportional to the squares of

the lengths of their sides. Write a formula stating this fact, letting u stand for the weight of a unit square, s stand for the length of a side of any square, and w for the weight of that square.

8. Let the unit square weigh $\frac{1}{2}$ pound and plot the graph of the formula obtained in Ex. 7.

9. From the graph in Ex. 8 find the approximate weights of squares of brass whose sides are 2, 4, 5 units respectively.

10. Write a formula and from it construct a "ready-reckoner" showing the price of pig-iron at $21.50 per ton.

11. Construct a ready-reckoner showing that a litre equals about 1.75 pints. How many pints, according to this graph, in $2\frac{1}{2}$, $3\frac{1}{3}$, 4 litres, respectively?

12. Construct $y \equiv x^2$, and determine from the graph $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{8}$, $\sqrt{10}$, $\sqrt{11}$, $\sqrt{15}$, approximately.

13. Construct $y \equiv 10^x$, and determine the values of y when $x = 1.5, -1, 1.9, -1.5, 2.5$.

IN THE SOLUTION OF PROBLEMS INVOLVING THE ELEMENT OF TIME

Many of the problems involving the element of time may be solved graphically. Those who have solved a sufficient number of the foregoing problems will need no further explanation to enable them to answer the following questions:—

EXERCISES

1. Call the shorter axis the time axis (TT') and the longer the rate axis (RR').

Plot the ready-reckoner showing the ground covered by a man whose rate is $3\frac{1}{2}$ miles per hour. (The formula used in this case $D \equiv TR$.) Suppose a second man, who had a handicap of 5 miles, travels at the rate of 3 miles per hour. What will represent his starting point? Where will he be at the end of three hours? At what point do the two ready-reckoners cross each other? What does this point tell you?

2. A steamboat running at the rate of 8 miles an hour sees a motorboat 10 miles off, going at the rate of 5 miles per hour. How far will the steamboat go before it overtakes the motorboat?

3. A travels 6 miles an hour and B 8 miles an hour. If A starts 3 hours before B, how long will B have to travel before he overtakes A? How far will they have travelled before this occurs?

4. Two cyclists, A and B, start out at the same time. A rides for $1\frac{1}{2}$ hours at a speed of 10 miles per hour, rests $\frac{1}{2}$ hour, and then continues on his course at 7 miles per hour. B rides without a stop at the rate of 8 miles per hour. How long before he overtakes A?

5. Two men start at the same time to walk around a circular course of 9 miles. The first man's rate is such that he completes the course once every $2\frac{1}{2}$ hours, and the second man's such that he completes it once every 3 hours. How long after starting will the second man pass the first? How long before he will pass him the second time?

(Hint: At what point will a man be when he has gone the course? How can this be shown using simply the pair of axes and no curved line?)

6. If from the same spot on a circular course of 2 miles two boys walk in the same direction at the rates of 5 and $3\frac{1}{2}$ miles an hour respectively, how often and at what intervals will they meet if they continued for 4 hours? If they walk in opposite directions how often and at what intervals will they meet?

7. A leaves town T and rows at the rate of $8\frac{1}{2}$ miles per hour to town T′ and back again. B leaves T′ at the same time that A leaves T, and rows at the rate of 7 miles per hour to T. Find the distance between T and T′, if A arives at town T 3 hours after B.

8. A train meets with an accident after travelling $1\frac{1}{2}$ hours. The accident delays it 2 hours, after which it travels at $\frac{3}{4}$ its former rate, and arrives at its destination 2 hours and 54 minutes late. If the accident had occurred 48 miles further on, the delay would have been 18 minutes less. How far had the train to run, and what were its rates before and after the accident?

9. A man rows 15 miles up a river and back again in 8 hours, rowing half again as fast with the stream as against it. What time did it take him to go up stream? What were his rates up and down?

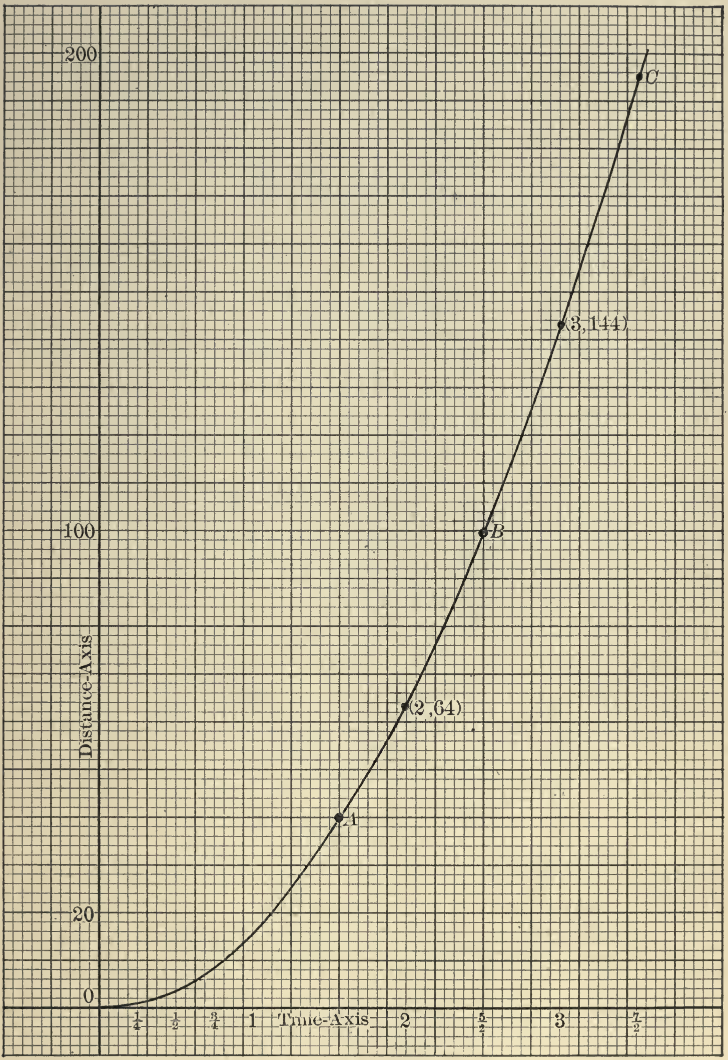
10. Two towns T and T′ are 60 miles apart. A walks from T to T′ at the rate of 3 miles per hour and trolleys back at the rate of 15 miles per hour. B starts from T′ 3 hours later than A from T, and drives to T at the rate of 6 miles an hour and walks back at the rate of 4 miles an hour. How long after starting and how far from T do they meet?

11. In how many years will the interest on $600 equal the amount on $200 if both are invested at 5%?

12. If one man invests $2,000 at 6%, and another invests $10,000 at 5%, in how many years will the amount of the first man's investment equal the interest on the second man's.

13. In how many years will the interest on $500 at 6% differ from the interest on $700 at 5% by $150?

14. Make various graphs which may be used in place of "interest tables."



Distance, in feet; scale, 1:2.

Time, in seconds; scale, 16:1.

Fig. 5. — Graph of the Formula $s \equiv 16\,t^2$.

CHAPTER III

STUDY OF THE FUNCTION AND THE EQUATION

The work we had in the preceding chapter in the graphic representation of formulas will help us to understand the following.

In the first place, when we consider the formula $C \equiv \pi d$, we see at once that whatever value we give to d, C will have a corresponding value. That is, as the formula now reads, C depends for its value upon the value given to d. In other words, the values of d and C may vary as much as we please, but once having fixed the value of d, that of C is also fixed. In this case both d and C are known as variable, but d is known as an independent variable and C as a dependent variable. If we were to solve the equation for d (that is, find $d \equiv C \div \pi$) which would be the dependent and which the independent variable? Why?

EXERCISES

1. Given a fixed principal and a fixed rate of interest, upon what variable would the amount of interest depend?

2. Ordinarily, upon what three variables does the amount of interest depend? Write a formula stating this.

3. Upon what two variables does the distance a man travels depend? Which are the independent and which the dependent variables in this case?

4. Give illustrations of independent and dependent variables in life—in nature.

Every dependent variable is known as a function of the independent variable or variables in question. For instance, we say that the amount of interest is a function of the independent variables, principal, time, and rate. Likewise, we say that $3x^2 + 5x + 6$ is a function of x, for it depends for its value upon the value given the variable x. This is usually written $f(x) \equiv 3x^2 + 5x + 6$. When $x = 2$, $f(x)$ becomes $f(2) \equiv 3(2)^2 + 5(2) + 6 \equiv 28$, and it is readily seen that as we give different values to x, $f(x)$ will have correspondingly different values.

Let us now call one axis the x-axis, and the other (say the vertical axis), the $f(x)$-axis, and attempt to plot the graphs of $f(x) \equiv 3x + 4$ in the following manner:—

I. Fill in the values omitted in the table:

Given $x =$	—5	—2	0	2	4
then $3x =$ and					
$f(x) \equiv 3x + 4 =$					

Thus we see that for each value given x, we have found a corresponding value for $f(x)$.

II. Plot the points representing these various pairs of values of x and $f(x)$.

III. Draw the graph determined by these points, and from it answer the following questions:—

a. What values of x produce a positive function?

b. For what values of x is the function negative?

c. If $x = -1.5$, what is the approximate value of $f(x)$?

EXERCISES

1. Given $f(x) \equiv x^2 + 5x - 7$.

(1) Fill in the values omitted in the following table:—

Given $x =$	—3	—2	—1	0	1	2	3	4	5	6	7
Then $x^2 =$											
and $5x =$											
and $f(x) \equiv x^2 + 5x - 7 =$											

(2) Plot the points found above, and draw as steady a line as you can through them.

(3) For what values of x does the function equal zero? 2? 3? 5? 10? —6?

(4) For what values of x is the function negative?

(5) For what values of x is the function positive?

(6) When $x = -2.5, +2.5$ what are the approximate values of $f(x)$?

(7) How many times does the graph cut the x-axis?

(8) How many factors has the expression $x^2 + 5x - 7$?

(9) What are they approximately?

(10) Could you find the factors exactly?

(11) If you were to plot the graph of $f(x) \equiv x^2 + 5x + 6$ where would you expect it to cut the x-axis?

2. By means of the method employed in the last exercise, plot the graph of:—

(1) $f(x) \equiv 3x^2 + 8x - 4$.

(2) $f(x) \equiv 4x^2 - 8x - 7$.

(3) $f(x) \equiv x^2 + 3x + 1$.

(4) $f(x) \equiv 3x^3 + 4x^2 - 8x - 7$.

3. Draw the graph of the parabola $f(x) \equiv x^2$ using values of x between $+$ and $-$ 5 inclusive.

4. Draw the graph of the circle $f(x) \equiv \pm\sqrt{36 - x^2}$. (Use integral values of x between $\pm$ 6 inclusive.)

5. Draw the graph of the ellipse $f(x) \equiv \pm \frac{1}{2}\sqrt{3(4 - x^2)}$.

6. Draw the graph of the hyperbola $f(x) \equiv \pm\sqrt{2x^2 + 7}$.

7. Draw the graph of $f(x) \equiv \frac{x^2}{4} - x + 2$.

Those of us who know the trigonometric ratios can now plot the graphs of functions containing them. One example will be sufficient to make this clear.

Given $x =$	0	$\frac{1}{6}\pi$ or 30°	$\frac{1}{4}\pi$ or 45°	$\frac{1}{3}\pi$ or 60°
$f(x) = \text{Sin} \equiv$	0	·5	$\frac{\sqrt{2}}{2} = .707$	$\frac{\sqrt{3}}{2} = .866$

$\frac{1}{2}\pi$ or 90°	$\frac{2}{3}\pi$ or 120°	$\frac{3}{4}\pi$ or 135°	π or 180°	$\frac{7}{6}\pi$ or 210°
1	$\frac{\sqrt{3}}{2}$ .866	$\frac{\sqrt{2}}{2}$ or .707	0	—.5

$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	2π
—.707	—.866	—1	—.866	—.707	0

—.5	—.707	etc.
$-\frac{1}{6}\pi$	$-\frac{1}{4}\pi$	etc.

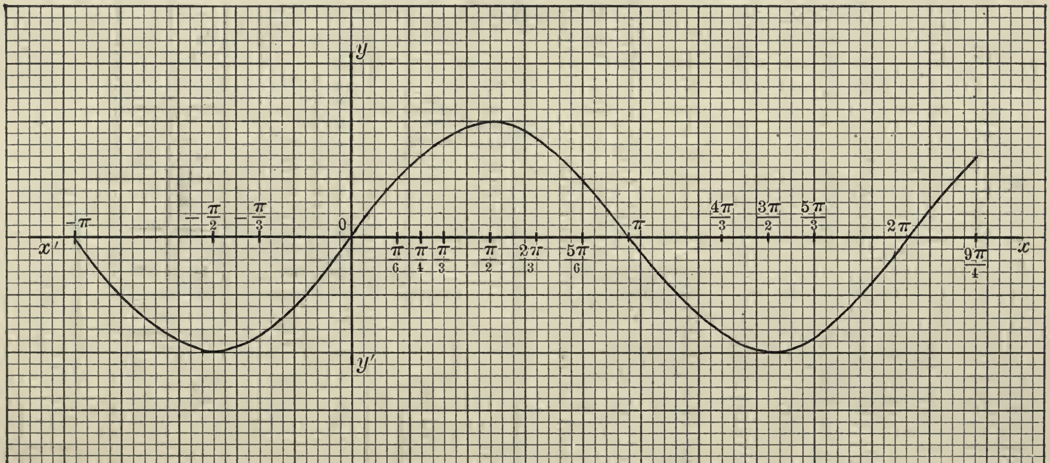


Fig. 6

Scale: xx', $24:\pi$. yy', $10:1$.

It is readily seen that $f(x) \equiv \sin x$ has as its limiting values $+1$ and -1. Therefore we shall use the shorter axis as the $f(x)$-axis, and the longer one as the x-axis. On the x-axis the unit π is divided into sixths, fourths, thirds, and halves, therefore we shall use 12 divisions to the unit on that axis (or a multiple of 12). In order to be able to measure tenths on the $f(x)$-axis we shall use 10 divisions to the unit on that axis. Finally, so that the graph may be more easily drawn, we shall use the scale 24 to π on the x-axis. Plotting the points found in the table we obtain the graph shown in Fig. 6.

NOTE.—Sin x is an example of what is called a periodic function—*i. e.*, a function which repeats the same values in the same order after a certain period. From the figure it is readily seen that $\sin(x + 360°)$ will be the same as $\sin x$. Therefore the period of $\sin x$ is $360°$ or 2π.

EXERCISES

1. If $\sin x = .7$ what will be the sine of (*a*) $(720° + x)$? (*b*) $(-360° + x)$?
2. Plot the graph of $\cos x \equiv f(x)$.
3. Plot the graph of $f(x) \equiv \tan x$.
4. Plot the graph of $f(x) \equiv \cot x$.
5. Plot the graph of $f(x) \equiv \sec x$.
6. Plot the graph of $f(x) \equiv \operatorname{cosec} x$.
7. Plot the graph of $f(x) \equiv \sin x + 2$.
8. Plot the graph of $f(x) \equiv \sin x + \cos x$.
9. Plot the graph of $f(x) \equiv \sin x - \cos x$.
10. Plot the graph of $f(x) \equiv 3 - \cos x$.
11. Are the above graphs those of periodic functions? If so, determine the period of each.

THE EQUATION

From what has been said in the beginning of this chapter it is easily seen that if $y = 3x + 4$, x would be the independent, and y the dependent variable and therefore a function of x. If then we call our axes xx' and yy' in place of x-axis and $f(x)$-axis, we may plot the graph of $y = 3x + 4$ just as above we plotted that of $f(x) \equiv 3x + 4$.

SINGLE LINEAR EQUATIONS

EXERCISES

1. Draw the graph of $y = 5x - \frac{1}{2}$, and from it find:
   (1) The value of x when $y = 0, 8, 10$.
   (2) The value of y when $x = 2, 1, -\frac{1}{2}$.

2. At what points will the line $y = 4x + 6$ cut the axes? What is the easiest way to find these points? What then is a simple way to plot an equation of the first degree? (Such equations are called *linear*.) Why?

3. Plot, by joining the points where the line cuts the axes:

(1) $y = x + 5$.
(2) $y = x - 5$.
(3) $y = -3x - 2$.
(4) $y = -3x + 2$.
(5) $x = -y + 4$.
(6) $5x + 2y = 7$.
(7) $9x + 7y - 8 = 0$.

4. Can you plot $x = -y$ by the method suggested in ex. 3? Give reason for your answer.

5. Plot (1) $x = -y$ (2) $x = 5$ (3) $y = -8$
(4) $x = 3y$ (5) $x = \frac{y}{4}$ (6) $x = y$

6. Give the equations stating that:
   (1) A point is always 10 units from a given line xx'.
   (2) A point is always 10 units from a line yy'.
   (3) A point is always at the same distance from each of two lines which intersect at right angles.

SIMULTANEOUS LINEAR EQUATIONS

7. On a single pair of axes draw the graphs of the following equations:

(1) $3x + 4y = 18$ (3) $\frac{3}{2}x - 9 = -2y$
(2) $5y - 2x = 11$ (4) $x + \frac{4}{3}y = 12$

8. From the graphs in Ex. 7 what can you say about equations (1) and (2)? (1) and (3)? (1) and (4)?

9. Two straight lines in the same plane in general intersect how often? May they do otherwise? Explain your answer.

10. What can you say of the equations of two straight lines whose graphs intersect once? What kind of equations must they be to give such result?

The line or group of lines that fulfills a given condition is termed the *locus* of that condition. For instance, the locus of the condition expressed in the equation $x = 3$ is the line drawn parallel to the yy' axis at a distance 3 units to the right of it.

Two loci are said to be coincident when every point in one lies on a corresponding point in the other, or in short, when they have all points in common. Two loci are said to be parallel when they have no point in common, and they are said to intersect when they have a finite number of points in common.

11. What can you say of the conditions expressed by (1) and (2), Ex. 7 above? by (1) and (3)? by (1) and (4)?

Two equations in the same variables are said to be consistent when they do not contradict each other, and inconsistent when they do.

12. Select pairs of consistent equations from Ex. 7.

13. Select pairs of inconsistent equations from Ex. 7.

14. From Ex. 7 can you tell whether all consistent equations can be solved simultaneously? Give a reason for your answer.

15. Do you suppose that inconsistent equations can be solved simultaneously?

16. How was equation (3), Ex. 7, derived from equation (1)? Are they consistent then? Would you say they were independent of each other?

17. How would you then define two consistent independent equations? Select two such equations from Ex. 7.

18. Arrange answers to the following questions just as the questions are arranged and underline the corresponding words and phrases in the two columns.

The Linear Equation	**The Straight Line**
1. A linear equation in two variables is satisfied by how many pairs of roots?	1. A straight line contains how many points?
2. The graph of a linear equation may be fixed by how many pairs of its roots?	2. The straight line is fixed by how many of its points?
3. In general two linear equations involving the same two variables have how many pairs of roots in common?	3. In general two coplanar straight lines have how many points in common?
4. May two linear equations in the same two variables have more than one pair of roots in common? What kind of equations are they then?	4. May two coplanar straight lines have more than one point in common? What kinds of lines are they?

5. May two linear equations in the same two variables have no pair of roots in common? What kind are such equations?

5. May two coplanar straight lines have no points in common? What kind of lines are they?

19. Solve the following equations graphically, using a new pair of axes for the solution of each pair:

(1) $\begin{cases} x - y = 2 \\ x + y = 8 \end{cases}$

(2) $\begin{cases} x + 2 = -2 \\ y = 2x \end{cases}$

(3) $\begin{cases} x + 2y = 7\frac{1}{2} \\ 2x + y = 7\frac{1}{2} \end{cases}$

(4) $\begin{cases} y - 25x = 13 \\ y + 62 = 50x \end{cases}$

(5) $\begin{cases} 5x + 2y = 8 \\ 2x - 3y = -12 \end{cases}$

SINGLE QUADRATIC EQUATION AND THOSE OF HIGHER DEGREE

Suppose we were now asked to solve the equation $x^2 + 5x + 6 = 0$. Factoring, we see at a glance that $(x + 3)(x + 2) = 0$, and therefore that $x = -3$ or -2.

Let us now see how we might have found these values by the graphic method. From what we have learned of functions of a variable and of the single linear equation we can readily plot the graph of $y = x^2 + 5x + 6$. Here we are not interested, however, in all the values of x, but just those which will make $y = 0$. Therefore, having drawn the graph of $f(x) \equiv x^2 + 5x + 6$ or $y \equiv x^2 + 5x + 6$, we run our eye along it until we find the points at which $y = 0$, or in short, at what points the graph cuts the

x-axis. At these points we find the values of x to be -2 and -3 if the graph is accurately drawn.

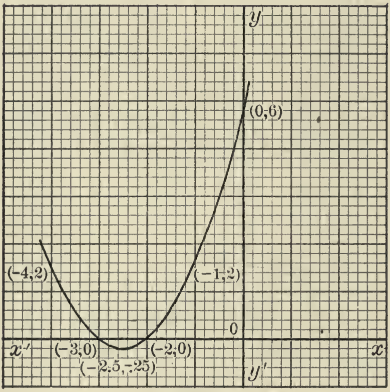


Fig.7

In a similar manner all quadratic equations—also those of higher degree—may be solved.

EXERCISES

1. Solve graphically the equations:

(1) $x^2 + 11x + 18 = 0.$
(2) $x^2 - 7x + 12 = 0.$
(3) $2x^2 + x + 1 = 0.$
(4) $4x^2 + 4x + 1 = 0.$
(5) $x^2 + x = 6.$
(6) $x^2 + 3 = 6x.$
(7) $9x^2 - 5x - 2 = 0.$
(8) $.9x^2 - 4.68x = -4.36.$
(9) $3x^3 + 10x^2 + 4.25x - 5 = 0.$
(10) $x^3 - 4.1x^2 - 1.05x + 11.025 = 0.$

2. How many times does the locus of a quadratic equation in x cut the x-axis?

3. Show graphically the character of the roots of the equations:

(1) $x^2 - 3x - 4 = 0$.

(2) $\frac{x^2}{4} - x + 2 = 0$. (Plot using values between $+3$ and -3.)

(3) $x^2 + 4x + 4 = 0$.

4. How does the graph of a quadratic equation indicate the fact that the roots of the equation are:

(1) Real and unequal?
(2) Real and equal?
(3) Imaginary?

SIMULTANEOUS LINEAR AND QUADRATIC EQUATIONS

Without any further preparation we may now solve the following sets of simultaneous equations.

EXERCISES

1. In what points does the straight line $3x + y = 25$ cut the circle $x^2 + y^2 = 65$?

2. The equation of a circle is $x^2 + y^2 = 49$, and the equation of a chord of the circle is $13x + 2y = 49$. Find the extremities of the chord.

3. Solve graphically the following pairs of equations:

(1) $\begin{cases} x^2 + y^2 = 10 \\ x = 3 \end{cases}$ (2) $\begin{cases} x^2 + y^2 = \frac{5}{2} \\ y = 3x - 5 \end{cases}$

(3) $\begin{cases} (x-1)^2 + (y-6)^2 = 25 \\ 4x + 3y + 3 = 0 \end{cases}$

Find the points common to the following parabolas and straight lines:

4. $y^2 = 9x$, $3x + 30 = 7y$.

5. $y^2 = 3x$, $x - 4y + 12 = 0$.

6. $y^2 = 4x$, $x = 6$, $y = -8$, $x = 0$, $x = -4$.

7. $y^2 = 8x, x + y = 6.$

8. $y^2 - 4x - 8y + 24 = 0, 3y - 2x = 8.$

Find the points of intersection of the following ellipses and straight lines:

9. $2x^2 + 3y^2 = 14, y - 2x = 0.$

10. $2x^2 + 3y^2 = 35, 4x + 9y = 35, 4x - 9y = 35.$

11. $9x^2 + 64y^2 = 576, 2y = x + 10, 2y = x + 1.$

Find the points common to the following hyperbolas and straight lines:

12. $x^2 - y^2 = 9, 4x + 5y = 40.$

13. $16x^2 - 9y^2 = 112, 9x + 16y = 100, 16x - 9y = 28.$

SIMULTANEOUS QUADRATIC EQUATIONS

Find approximately the points of intersection of the following loci:

14. $2x^2 + 3y^2 = 14, y^2 = 4x.$

15. $x^2 + y^2 = 10, x^2 + 7y^2 = 16.$

16. $x^2 + y^2 = 25, xy = 5.$

MISCELLANEOUS EXERCISES

1. Find the two square roots of 6.
(Hint: Plot the graph of $f(x) \equiv x^2$.)

2. Find the three cube roots of 8. $f(x) \equiv x^3$.

3. Find the six sixth roots of 1.

4. Which of the above roots cannot be shown graphically?

5. Write the equations of two parallel lines and construct them.

6. Write the general equations of two parallel lines.

7. The equation of the circle $ax^2 + ay^2 = C$ differs in what respect from the equation of the ellipse $ax^2 + by^2 = C$? What is the shape of the ellipse when a and b differ

greatly in value? When a and b are nearly equal? When a and b are equal?

8. Draw a graph by means of which American money may be changed to:—

(1) English money. (3) French money.
(2) German money.

9. Solve graphically $\begin{cases} x^2 + xy + y^2 = 7 \\ x - y = 1. \end{cases}$

10. Two bodies 140 feet apart move towards each other, the first at the rate of 10 feet per second, the second four-fifths as fast. How long before they are 44 feet apart?

APPENDIX TO CHAPTER II

Draw graphs to represent the statistics given in the following tables:

1. The monthly mean maximum temperature Fahrenheit in the cities noted for the years 1872 to 1901:

	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Alpena, Mich.	26	26	32	46	59	69	75	73	66	53	39	30
Boston, Mass.	35	36	42	54	66	76	81	78	71	60	49	39
Buffalo, N. Y.	31	31	37	50	62	72	77	76	70	58	45	36
Chicago, Ill.	31	33	41	54	64	74	80	78	72	60	45	36
Cincinnati, Ohio	40	43	51	63	74	83	87	84	78	66	52	43
Cleveland, Ohio	33	35	41	54	66	76	80	78	72	61	47	38
Key West, Fla.	74	76	77	80	84	87	89	89	87	83	78	74
La Crosse, Wis.	24	28	39	57	69	78	83	80	71	59	41	30
Montgomery, Ala.	57	61	67	76	84	90	92	90	86	76	66	58
New York, N. Y.	37	38	44	57	68	78	82	80	74	63	51	41
Norfolk, Va.	48	51	56	65	75	84	88	85	79	69	59	51
Oswego, N. Y.	31	31	37	50	63	73	78	76	70	57	45	36
St. Paul, Minn.	20	24	36	56	68	77	83	80	71	57	38	27

2. The monthly mean minimum temperature Fahrenheit in the cities noted for the years 1872 to 1901:

	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Alpena, Mich.	12	10	16	31	41	52	57	55	49	39	28	19
Boston, Mass.	19	20	27	37	48	58	63	62	55	45	34	24
Buffalo, N. Y.	18	17	24	35	46	58	63	61	55	44	33	24
Chicago, Ill.	16	19	27	39	49	59	65	65	58	46	32	23
Cincinnati, Ohio	25	27	34	45	56	65	69	67	60	48	37	30
Cleveland, Ohio	20	20	27	38	50	59	64	62	56	45	34	25
Key West, Fla.	65	67	68	71	75	78	79	79	78	75	71	66
La Crosse, Wis.	7	10	22	38	50	60	64	61	53	41	26	15
Montgomery, Ala.	39	43	48	55	63	70	73	72	67	56	46	40
New York, N. Y.	24	24	30	40	52	61	67	66	60	48	38	28
Norfolk, Va.	33	35	39	47	58	66	71	70	65	54	44	36
Oswego, N. Y.	17	17	24	36	46	56	62	61	54	43	33	22
St. Paul, Minn.	2	7	18	36	48	58	62	60	51	39	22	11

The preceding material as well as what follows should be made use of in various ways as may be suggested by both pupils and teacher. For instance, on a single sheet of cross-section paper make a diagram showing the mean maximum and the mean minimum temperatures of Baltimore, Md., using a dotted line to show the mean maximum, and a solid line to show the mean minimum. Using red ink draw a line showing the probable mean temperature.

3. Average amounts of precipitation for the year 1904:

	San Francisco, Cal.	Atlanta, Ga.	Lincoln, Neb.	Santa Fe, New Mex.	Salt Lake City, Utah	Yellowstone Park, Wyo.
Jan......	4.75	5.2	.67	.58	1.33	2.4
Feb......	3.31	4.02	.87	.74	1 4	1.92
March...	3.23	5.94	1.21	.71	1.99	2.3
April....	1.80	3.69	2.67	.75	2.13	1.23
May.....	.41	3.26	4.59	1.15	1.97	1.94
June.....	.19	4.03	4.36	1.04	.73	1.65
July.....	.02	4.86	4.13	2.7	.52	1.23
Aug......	.01	4.52	3.39	2.43	.74	1.07
Sept.....	.44	3.55	2.14	1.64	.80	.99
Oct......	1.32	2.26	2.07	1.05	1.5	1.09
Nov......	2.70	3.44	.77	.68	1.4	1.59
Dec......	4.21	4.35	.76	.72	1.43	1.86

4. The population of New York City to the nearest 1,000 for the years indicated:

YEAR	POPULATION	YEAR	POPULATION	YEAR	POPULATION
1790.....	33,000	1830.....	203,000	1870.....	942,000
1800.....	60,000	1840.....	313,000	1880.....	1,206,000
1810.....	96,000	1850.....	516.000	1890.....	1,515,000
1820.....	124,000	1860.....	806,000	1900.....	3,437,000*

* All Boroughs.

Plot the above correct to 10,000 only.

5. Immigration into the United States, correct to the nearest 1,000:

YEAR	IMMIGRANTS	YEAR	IMMIGRANTS	YEAR	IMMIGRANTS
1820.....	8,000	1860.....	133,000	1890.....	455,000
1825.....	10,000	1862.....	72,000	1892.....	623,000
1830.....	23,000	1865.....	180,000	1898.....	229,000
1835.....	45,000	1870.....	387,000	1900.....	449,000
1840.....	84,000	1875.....	227,000	1902.....	649,000
1845.....	114,000	1880.....	457,000	1903.....	857,000
1850.....	370,000	1882.....	789,000	1904.....	813,000
1855.....	201,000	1885.....	395,000		

6. Income and Expenditures of the United States Government, 1876–1905. (Record to the nearest $1,000,000):

YEAR	REVENUE	EXPENDITURES
1876...........	$287,482,039	$258,459,797
1880...........	333,526,611	267,642,958
1885...........	323,690,706	260,226,935
1890...........	403,080,983	318,040,711
1895...........	313,390,075	356,195,298
1900...........	567,240,852	487,713,792
1905...........	543,423,859	567,411,611

7. Public Schools in the United States:

YEAR	Population 5 to 18 years of age (in millions)	Expenditures per capita of this population (in dollars)	YEAR	Population 5 to 18 years of age (in millions)	Expenditures per capita of this population (in dollars)
1871..........	12.3	5.62	1895..........	20.4	8.60
1876..........	13.7	6.06	1899..........	21.9	9.13
1880..........	15.1	5.17	1900..........	21.4	10.04
1885..........	16.7	6.61	1905..........	23.4	12.46
1890..........	18.5	7.60	1906..........	23.8	12.94

8.

	1871	1873	1875	1877	1880	1885	1888	1892	1896	1900	1907
Amount of money in the U. S. July 1 (in dollars)	18.75	18.58	18.16	16.46	23.64	27.38	28.20	26.92	25.62	30.66	36.30
Debt less cash in Treasury July 1 (in dollars)	56.81	50.52	47.53	43.56	38.27	24.50	17.72	12.93	13.60	14.52	10.22
Merchandise imported for consumption per capita (in dollars)	12.65	15.91	11.97	9.49	12.51	10.32	11.88	12.50	10.81	10.88	16.49
Total exports of Domestic Merchandise per capita (in dollars)	10.83	12.12	11.36	12.72	16.43	12.94	11.40	15.61	12.29	17.96	21.60
Retained for consumption per capita: Raw Cotton (in pounds)	14.10	15.19	11.90	14.03	18.94	15.16	19.59	24.58	18.67	22.57	29.53
Retained for consumption per capita: Wheat and Wheat Flour (in bushels)	4.69	4.81	5.38	5.01	5.35	6.77	5.62	5.94	4.85	4.74	6.86
Retained for consumption per capita: Corn and Corn Meal (in bushels)	27.40	22.86	18.66	26.13	28.88	31.04	23.86	30.48	29.18	24.44	33.11
Retained for consumption per capita: Sugar (in pounds)	36.2	39.8	43.6	38.9	42.9	51.8	56.7	63.8	62.5	65.2	77.5
Retained for consumption per capita: Coffee (in pounds)	7.91	6.87	7.08	6.94	8.78	9.60	6.81	9.67	8.11	9.81	11.36
Imports and Exports of Merchandise by sea carried in American vessels (in per cent.)	31.9	26.4	26.2	26.9	17.4	15.3	14.	12.3	12.	9.3	10.6
Post-Office Department: Revenue per capita (in dollars)	.51	.55	.61	.59	.66	.76	.88	1.09	1.17	1.34	2.13
Post-Office Department: Expenditures per capita (in dollars)	.62	.70	.79	.72	.73	.89	.94	1.19	1.34	1.46	2.25

9. Density of population per square mile, of States and Territories, 1790–1900:

YEAR	Connecticut	Delaware	Georgia	Kentucky	Maine	Massachusetts	New Hampshire	New York	North Carolina	Pennsylvania	Rhode Island
1790	49.1	30.2	1.4	1.8	3.2	47.1	15.8	7.1	8.1	9.7	63.4
1800	51.8	32.8	2.8	5.5	5.1	52.6	20.4	12.4	9.8	13.4	63.7
1810	54.1	37.1	4.3	10.2	7.7	58.7	23.8	20.1	11.4	18.0	70.9
1820	56.8	37.1	5.8	14.1	10.0	65.1	27.1	28.8	13.2	23.3	76.6
1830	61.4	39.2	8.8	17.2	13.4	75.9	29.9	40.3	15.2	30.0	89.6
1840	64.0	39.8	11.7	19.5	16.8	91.8	31.6	51.0	15.5	38.3	100.3
1850	76.5	46.7	15.4	24.6	19.5	123.7	35.3	65.0	17.9	51.4	136.0
1860	95.0	57.3	17.9	28.9	21.0	153.1	36.2	81.5	20.4	64.6	160.9
1870	110.9	63.8	20.1	33.0	21.0	181.3	35.3	92.0	22.1	78.3	200.3
1880	128.5	74.8	26.1	41.2	21.7	221.8	38.5	106.7	28.8	95.2	254.9
1890	154.0	86.0	31.2	46.5	22.1	278.5	41.8	126.1	33.3	116.9	318.4
1900	187.5	94.3	37.6	53.7	23.2	348.9	45.7	152.6	39.0	140.1	407.0

10. Native and Foreign born population of various cities, correct to the nearest 100:

CITY	1870	1880	1890	1900
Washington, D. C.:				
Native born	95,400	133,100	211,600	258,600
Foreign born	13,800	14,200	18,800	20,100
Buffalo, N. Y.:				
Native born	71,500	103,900	166,200	248,100
Foreign born	46,200	51,300	89,500	104,300
San Francisco, Cal.:				
Native born	75,800	129,800	172,200	225,900
Foreign born	73,800	104,200	126,800	116,900
Portland, Oreg.:				
Native born	5,700	11,300	29,100	64,600
Foreign born	2,600	6,300	17,300	25,900
Atlanta, Ga.:				
Native born	20,700	36,000	63,700	87,300
Foreign born	1,100	1,400	1,900	2,500
Savannah, Ga :				
Native born	24,600	27,700	39,800	50,800
Foreign born	3,700	3.000	3,400	3,400
Hoboken, N. J.:				
Native born	10,000	18,000	26,300	38,000
Foreign born	10,300	13,000	17,400	21,400

11. The population of a few States, by color at each census:

YEAR	MAINE White	MAINE Colored	SOUTH CAROLINA White	SOUTH CAROLINA Colored	GEORGIA White	GEORGIA Colored
1790.....	96,002	538	140,178	108,895	52,886	29,662
1800.....	150,901	818	196,255	149,336	102,261	60,425
1810.....	227,736	969	214,196	200,919	145,414	107,019
1820.....	297,406	929	237,440	265,301	189,570	151,419
1830.....	398,263	1,192	257,863	323,322	296,806	220,017
1840.....	500,438	1,355	259,084	335,314	407,695	283,697
1850.....	581,813	1,356	274,563	393,944	521,572	384,613
1860.....	626,952	1,327	291,388	412,320	591,588	465,698
1870.....	625,309	1,606	289,792	415,814	638,967	545,142
1880.....	647,485	1,451	391,245	604,332	817,047	725,133
1890.....	659,896	1,190	462,215	688,934	978,538	858,815
1900.....	693,147	1,319	557,995	782,321	1,181,518	1,034,813

12. The areas of Indian Reservations for the years indicated given in square miles:

YEAR	ARIZONA	IOWA	NEBRASKA	N. CAROLINA
1880.....	4,832.5	1	682	102
1890. ...	10,317.5	2	214	102
1900.....	23,673	4.5	116	153.5
1907.....	26,532.7	4.63	23.08	98.77

13. Departures of passengers from seaports of the United States for foreign countries 1868 to 1907, correct to the nearest 100:

YEAR	TOTAL	YEAR	TOTAL	YEAR	TOTAL
1868.....	32,500	1879.....	51,400	1898.....	94,600
1870.....	33,600	1885.....	87,800	1900.....	155,900
1872.....	39,900	1890.....	105,900	1905.....	201,200
1873.....	52,100	1891.....	107,100	1907.....	224,900
1876.....	46,400	1893.....	95,100		
1878.....	55,200	1894.....	121,900		

14. Records of Cereal Crops, 1866 to 1907:

YEAR	WHEAT—Average		OATS—Average		BARLEY—Average	
	Per acre	Value per acre Dec. 1	Per acre	Value per acre Dec. 1	Per acre	Value per acre Dec. 1
	Bushels	*Dollars*	*Bushels*	*Dollars*	*Bushels*	*Dollars*
1866.....	9.9	15.05	30.2	10.61	22.9	16.07
1867.....	11.6	16.83	25.9	11.53	22 7	15.94
1868.....	12.1	13.17	26.4	11.00	24.4	26.61
1869.....	13.6	10.38	30.5	11.58	27.9	19.79
1870.....	12.4	11.73	28.1	10.97	23.7	18.75
1871.....	11.6	13.24	30.6	11.07	24.0	18.19
1872.....	11.9	13.35	30.2	9.03	19.2	13.18
1873.....	12.7	13.56	27.7	9.59	23.1	20.04
1874.....	12.3	10.65	22.1	10.38	20.6	17.71
1875.....	11.1	9.91	29.7	9.52	20.6	15.29
1876. ..	10.5	10.09	24.0	7.77	21.9	13.81
1877.....	13.9	14.65	31.7	9.01	21.3	13.40
1878.....	13.1	10.15	31.4	7.72	23.6	13.66
1879.....	13 8	15.27	28.7	9.50	24.0	14.11
1880.....	13.1	12.48	25.8	9.28	24.5	16.32
1881.....	10.2	12.12	24.7	11.48	20.9	17.21
1882.....	13.6	12.02	26.4	9.89	21.5	13.54
1883.....	11.6	10.52	28.1	9.20	21.1	12.37
1884.....	13.0	8.38	27.4	7.58	23.5	11.41
1885.....	10.4	8.05	27.6	7.88	21.4	12.04
1886.....	12.4	8.54	26.4	7.87	22.4	12.00
1887.....	12.1	8.25	25.4	7.74	19.6	10.15
1888.....	11.1	10.32	26.0	7.24	21.3	12.57
1889.....	12.9	8.98	27.4	6.26	24.3	10.13
1890.....	11.1	9.28	19.8	8.40	21.4	13.44
1891.....	15.3	12.86	28.9	9.08	25.9	13.56
1892.....	13.4	8.35	24.4	7.73	23.6	11.18
1893.....	11.4	6.16	23.4	6.88	21.7	8.92
1894.....	13.2	6.48	24.5	7.95	19.4	8.56
1895.....	13.7	6.99	29.6	5.87	26.4	8.88
1896.....	12.4	8.97	25.7	4.81	23.6	7.62
1897.....	13 4	10.86	27.2	5.75	24.5	9.25
1898.....	15.3	8.92	28.4	7.23	21.6	8.93
1899.....	12.3	7.17	30.2	7.52	25.5	10.28
1900.....	12.3	7.61	29.6	7.63	20.4	8.32
1901.....	15.0	9.37	25.8	10.29	25.6	11.57
1902.....	14.5	9.14	34.5	10.60	29.0	13.28
1903.....	12.9	8.96	28.4	9.68	26.4	12.05
1904.....	12.5	11.58	32.1	10.05	27.2	11.40
1905.....	14 5	10 83	34.0	9.88	26.8	10.80
1906.....	15.5	10.37	31.2	9.89	28.3	11.74
1907.....	14.0	12.26	23.7	10.51	23.8	15.86

15. Value of gold and silver produced in the United States. (Plot correct to the half-million dollars, showing on separate sheets the gold and silver production, and on one sheet the amount of gold produced in California, other States and Territories, and the total amount produced.)

YEAR	GOLD			SILVER
	California	Other States and Territories	Total	
	Dollars	*Dollars*	*Dollars*	*Dollars*
1860	45,000,000	1,000,000	46,000,000	156,800
1861	40,000,000	3,000,000	43,000,000	2,062,000
1862	34,700,000	4,500,000	39,200,000	4,684,800
1863	30,000,000	10,000,000	40,000,000	8,842,300
1864	26,600,000	19,500,000	46,100,000	11,443,000
1865	28,500,000	24,725,000	53,225,000	11,642,200
1866	25,500,000	28,000,000	53,500,000	10,356,400
1867	25,000,000	26,725,000	51,725,000	13,866,200
1868	22,000,000	26,000,000	48,000,000	12,306,900
1869	22,500,000	27,000,000	49,500,000	12,297,600
1870	25,000,000	25,000,000	50,000,000	16,434,000
1871	20,000,000	23,500,000	43,500,000	23,588,300
1872	19,000,000	17,000,000	36,000,000	29,396,400
1873	17,000,000	19,000,000	36,000,000	35,881,600
1874	17,500,000	15,990,900	33,490,900	36,917,500
1875	17,617,000	15,850,900	33,467,900	30,485,900
1876	17,000,000	22,929,200	39,929,200	34,919,800
1877	15,000,000	31,897,400	46,897,400	36,991,500
1878	15,300,000	35,906,400	51,206,400	40,401,000
1879	16,000,000	22,900,000	38,900,000	35,477,100
1880	17,500,000	18,500.000	36,000,000	34,717,000
1881	18,200,000	16,500,000	34,700,000	37,657,500
1882	16,800,000	15,700,000	32,500,000	41,105,900
1883	14,120,000	15,880,000	30,000,000	39,618,400
1884	13,600,000	17,200,000	30,800,000	41,921,300
1885	12,700,000	19,101,000	31,801,000	42,503,500
1886	14,725,000	20,144,000	34,869,000	39,482,400
1887	13,400,000	19,736,000	33,136,000	40,887,200
1888	12,750,000	20,417,500	33,167,500	43,045,100
1889	13,000,000	19,967,000	32,967,000	46,838,400

15. Value of Gold and Silver Produced in the United States—*Continued.*

Year	Gold: California	Gold: Other States and Territories	Gold: Total	Silver
	Dollars	*Dollars*	*Dollars*	*Dollars*
1890	12,500,000	20,345,000	32,845,000	57,242,100
1891	12,600,000	20,575,000	33,175,000	57,630,000
1892	12,000,000	21,015,000	33,015,000	55,662,500
1893	12,080,000	23,875,000	35,955,000	46,800,000
1894	13,570,000	25,930,000	39,500,000	31,422,100
1895	14,929,000	31,681,000	46,610,000	36,445,500
1896	15,235,900	37,852,400	53,088,000	39,654,600
1897	14,618,300	42,744,700	57,363,000	32,316,000
1898	15,637,900	48,825,100	64,463,000	32,118,400
1899	15,197,800	55,855,600	71,053,400	32,859,000
1900	15,816,200	63,354,800	79,171,000	35,741,140

16. Anthracite and bituminous coal production in the United States. (Show record on a single pair of axes and correct to one million.)

Year	Total Anthracite	Total Bituminous	Year	Total Anthracite	Total Bituminous
	Tons	*Tons*		*Tons*	*Tons*
1880	25,580,189	38,242,641	1901	60,302,264	201,572,572
1890	41,489,858	99,377,073	1902	37,024,582	232,252,596
1897	47,036,389	131,739,681	1903	66,678,392	252,389,837
1898	47,705,125	148,702,257	1904	65,382,842	248,738,941
1899	54,030,536	172,524,099	1905	69,405,958	281,239,252
1900	51,309,214	189,480,097			

17. Number of employees thrown out of work because of strikes. Correct to nearest hundred. (Plot correct to 1,000.)

Year	Number	Year	Number	Year	Number
1881	129,500	1890	352,000	1899	417,100
1882	154,700	1891	299,000	1900	505,100
1883	149,800	1892	206,700	1901	543,400
1884	147,100	1893	265,900	1902	659,800
1885	242,700	1894	660,400	1903	656,100
1886	508,000	1895	392,400	1904	517,200
1887	379,700	1896	241,200	1905	221,700
1888	147,700	1897	408,400		
1889	249,600	1898	249,000		

18. Number of strikes:

CALENDAR YEAR	Ordered by labor organizations	Not ordered by labor organizations	CALENDAR YEAR	Ordered by labor organizations	Not ordered by labor organizations
1881.....	223	248	1894.....	847	501
1882.....	220	234	1895.....	658	555
1883.....	271	207	1896.....	662	363
1884.....	240	203	1897.....	596	482
1885.....	357	288	1898.....	638	418
1886.....	763	669	1899.....	1,115	682
1887.....	952	483	1900.....	1,164	615
1888.....	616	288	1901.....	2,218	706
1889.....	724	351	1902.....	2,474	688
1890.....	1,306	525	1903.....	2,754	740
1891.....	1,284	432	1904.....	1,895	412
1892.....	918	380	1905.....	1,552	525
1893.....	906	399			

19. Number of Post Offices in the United States, correct to the nearest 500.

YEAR ENDED JUNE 30TH	POST OFFICES	YEAR ENDED JUNE 30TH	POST OFFICES
1879........	41,000	1894........	70,000
1880........	43,000	1895........	70,000
1881........	44,500	1896........	70,500
1882........	46,000	1897........	71,000
1883........	48,000	1898........	73,500
1884........	50,000	1899........	75,000
1885........	51,500	1900........	76,500
1886........	53,500	1901........	77,000
1887........	55,000	1902........	76,000
1888........	57,500	1903........	74,000
1889........	59,000	1904........	71,000
1890........	62,500	1905........	68,000
1891........	64,500	1906........	65,500
1892........	67,000	1907........	62,500
1893........	68,500		

20. Number of offices of the Postal Telegraph Cable Company, correct to the nearest 100.

YEAR	OFFICES	YEAR	OFFICES	YEAR	OFFICES	YEAR	OFFICES
1885..	300	1891..	1,200	1897..	9,900	1903..	20,000
1886..	400	1892..	1,400	1898..	11,100	1904..	21,100
1887..	600	1993..	1,600	1899..	12,700	1905..	23,100
1888..	700	1894..	1,800	1900..	13,100	1906..	25,300
1889..	800	1895..	2,100	1901..	14,900	1907..	25,500
1890..	1,000	1896..	9,100	1902..	16,200		

21. Table showing the increase in mileage of railroad in operation in the United States. Given correct to nearest unit:

YEAR	New England	Middle Atlantic	Central Northern	South Atlantic	Gulf and Mississippi Valley	Southwestern	Northwestern	Pacific	GRAND TOTAL
1860..	3,660	6,353	9,583	5,463	3,727	1,162	655	23	30,626
1870..	4,494	10,577	14,701	6,481	5,106	4,625	5,004	1,934	52,922
1880..	5,982	15,147	25,109	8,474	6,995	14,085	12,347	5,128	93,267
1890..	6,832	20,038	36,976	17,301	13,343	32,888	27,294	12,031	166,703
1900..	7,501	22,385	41,138	21,905	16,211	37,530	32,106	15,486	194,262
1904..	7,619	23,150	43,252	23,589	18,297	44,852	34,307	17,328	212,394
1905..	7,681	23,408	43,959	24,180	19,026	46,061	35,157	17,869	217,341
1906..	7,729	23,559	44,427	24,897	19,735	47,447	36,097	18,743	222,634

22. Average receipts per ton per mile on leading railroads of the United States:

YEAR	CENTS	YEAR	CENTS
1870........	4.50	1903........	.98
1880........	2.21	1904........	.99
1890........	1.50	1905........	.94
1900........	.93	1906........	.93
1902........	1.01		

23. Number of persons killed by railway accidents in the United States, 1888 to 1906:

YEAR ENDED JUNE 30TH	EMPLOYEES	PASSENGERS	OTHER PERSONS
1888	2,070	315	2,897
1889	1,972	310	3,541
1890	2,451	286	3,598
1891	2,660	293	4,076
1892	2,554	376	4,217
1893	2,727	299	4,320
1894	1,823	324	4,300
1895	1,811	170	4,155
1896	1,861	181	4,406
1897	1,693	222	4,522
1898	1,958	221	4,680
1899	2,210	239	4,674
1900	2,550	249	5,066
1901	2,675	282	5,498
1902	2,669	345	5,274
1903	3,606	355	5,879
1904	3,632	441	5,973
1905	3,361	537	5,805
1906	3,929	359	6,330

24. Table showing the number of sailing and steam vessels in use in the United States, correct to nearest 100:

YEAR ENDED JUNE 30TH	SAILING VESSELS	STEAM VESSELS
1879	20,600	4,600
1884	18,700	5,400
1889	17,700	5,900
1894	17,100	6,500
1899	15,900	6,800
1902	16,500	7,700
1907	14,900	10,100

25. Comparison of the number of various kinds of vessels built in the United States, 1881–1907:

YEAR ENDED JUNE 30TH	SAILING VESSELS				STEAM VESSELS			Canal boats	Barges	TOTAL
	Ships and Barks	Brigs	Schooners	Sloops	Sidewheel	Stern-wheel	Propeller			
1881	29	3	318	143	55	105	284	57	114	1,108
1883	33	2	567	119	46	90	303	42	66	1,268
1884	24	2	533	147	32	103	275	33	41	1,190
1885	11	....	379	143	39	86	213	21	28	920
1886	8	1	276	120	18	80	142	23	47	715
1887	7	1	258	181	24	69	206	36	62	844
1888	4	....	275	144	33	84	313	40	121	1,014
1889	1	....	296	192	28	87	325	88	60	1,077
1890	10	....	347	148	26	99	285	40	96	1,051
1891	13	1	447	272	28	111	349	57	106	1,384
1892	8	....	423	415	26	105	307	37	74	1,395
1893	8	1	303	181	19	93	268	28	55	956
1894	3	....	253	221	26	61	206	14	54	838
1895	1	....	188	208	17	70	161	11	38	694
1896	2	....	215	152	25	84	177	13	55	723
1897	1	....	160	177	20	88	180	70	195	891
1898	1	....	159	199	15	170	209	20	179	952
1899	3	....	223	194	14	182	243	13	401	1,273
1900	4	....	281	219	19	117	286	38	483	1,447
1901	6	....	259	261	21	131	354	79	469	1,580
1902	9	....	316	256	27	137	415	44	287	1,491
1903	3	....	298	169	28	131	392	19	271	1,311
1904	....	....	203	127	13	161	439	25	216	1,184
1905	....	....	195	115	10	164	386	30	202	1,102
1906	....	....	154	75	16	147	487	83	259	1,221
1907	....	....	81	66	15	149	510	62	274	1,157

26. Lives lost through disasters to vessels on rivers of the United States:

YEAR	LIVES LOST	YEAR	LIVES LOST	YEAR	LIVES LOST
1887	89	1894	29	1901	19
1888	17	1895	15	1902	157
1889	78	1896	50	1903	35
1890	63	1897	7	1904	30
1891	129	1898	25	1905	20
1892	50	1899	41	1906	34
1893	34	1900	18	1907	24

27. Table showing some work performed by Revenue Cutter Service.

	1901	1902	1903	1904	1905	1906	1907
Lives saved (actually rescued) from drowning....	178	55	19	24	18	17	41
Persons in distress taken on board and cared for..	101	538	31	47	187	1,285	78
Vessels assisted...........	107	101	71	154	521	131	138
Vessels seized or reported for violation of law......	178	191	230	494	262	378	319

28. Table showing total amount of merchandise imported into and exported from the United States. (Correct to the nearest million):

Year ended June 30th	TOTAL VALUE IMPORTS	TOTAL VALUE EXPORTS	Year ended June 30th	TOTAL VALUE IMPORTS	TOTAL VALUE EXPORTS
	Million Dollars	*Million Dollars*		*Million Dollars*	*Million Dollars*
1870....	436	377	1889....	745	730
1871....	520	428	1890....	789	845
1872....	627	428	1891....	845	872
1873....	642	505	1892....	827	1,016
1874....	567	569	1893....	866	831
1875....	533	499	1894....	655	869
1876....	461	526	1895....	732	793
1877....	451	590	1896....	780	863
1878....	437	681	1897....	765	1,032
1879....	446	698	1898....	616	1,210
1880....	668	824	1899....	697	1,204
1881....	643	884	1900....	850	1,371
1882....	725	783	1901....	823	1,460
1883....	723	804	1902....	903	1,355
1884....	668	725	1903....	1,026	1,392
1885....	578	727	1904....	991	1,435
1886....	635	666	1905....	1,118	1,492
1887....	692	703	1906....	1,227	1,718
1888....	724	684	1907....	1,434	1,854

29. Table showing value of exports of cotton goods of domestic manufacture. (Correct to nearest million):

YEAR	MILLION DOLLARS	YEAR	MILLION DOLLARS	YEAR	MILLION DOLLARS
1856....	7	1874....	3	1892....	13
1857....	6	1875....	4	1893....	12
1858....	6	1876....	8	1894....	14
1859....	8	1877....	10	1895....	14
1860....	11	1878....	11	1896....	17
1861....	8	1879....	11	1897....	21
1862....	3	1880....	10	1898....	17
1863....	3	1881....	14	1899....	24
1864....	1	1882....	13	1900....	24
1865....	3	1883....	13	1901....	20
1866....	2	1884....	12	1902....	32
1867....	5	1885....	12	1903....	32
1868....	5	1886....	14	1904....	22
1869....	6	1887....	15	1905....	50
1870....	4	1888....	13	1906....	53
1871....	4	1889....	10	1907....	32
1872....	2	1890....	10		
1873....	3	1891....	14		

30. Annual average price in dollars per ton of coal:

YEAR	ANTHRACITE	BITUMINOUS	YEAR	ANTHRACITE	BITUMINOUS
1850....	3.64	....	1870....	4.39	4.72
1853....	3.70	3 30	1875....	4.39	4.35
1855....	4.49	3.89½	1877....	2.59	3.15
1860 ...	3.40	3.49	1880....	4.53	3.75
1861....	3.39	3.44	1885....	4.10	2.25
1862....	4.14	4.23	1890....	3.92½	2.60
1863....	6.06	5.57	1895....	3.50	2.00
1864....	8.39	6.84	1898....	3.50	1.60
1865....	7.86	7.57	1900....	3.47	2.50
1866....	5.80	5.94	1905....	4.50	2.60

31. Value of sugar and molasses imported into the United States. (To the nearest half million):

Year ended June 30th	SUGAR	MOLASSES	Year ended June 30th	SUGAR	MOLASSES
	Dollars in Millions	*Dollars in Millions*		*Dollars in Millions*	*Dollars in Millions*
1861....	30.5	4.0	1885....	72.5	4.0
1862....	20.5	3.5	1886....	81.0	5.5
1863....	19.0	4.5	1887....	78.5	5.5
1864....	29.5	7.5	1888....	74.0	5.5
1865....	27.5	7.5	1889....	88.5	5.0
1866....	40.5	7.5	1890....	96.0	5.0
1867....	36.0	11.5	1891....	106.0	2.5
1868....	49.5	12.0	1892....	104.5	3.0
1869....	60.5	12.0	1893....	116.5	2.0
1870....	57.0	13.0	1894....	127.0	2.0
1871....	64.5	10.0	1895....	76.5	1.5
1872....	81.0	10.5	1896....	89.0	.5
1873....	82.5	10.0	1897....	99.0	.5
1874....	82.0	11.0	1898....	60.5	.5
1875....	73.5	11.5	1899....	95.0	1.0
1876....	58.0	8.0	1900....	101.0	1.0
1877....	85.0	8.0	1901....	90.5	1.0
1878....	73.0	7.0	1902....	55.0	1.0
1879....	72.0	7.0	1903....	72.0	1.0
1880....	80.0	8.5	1904....	72.0	1.0
1881....	86.5	6.5	1905....	97.5	1.0
1882....	90.5	10.0	1906....	85.5	.5
1883....	91.5	7.5	1907....	93.0	1.0
1884....	98.0	5.5			

32. Average food cost per workingman's family in the United States, 1890–1906:

YEAR	United States, 2,567 families	YEAR	United States, 2,567 families	YEAR	United States, 2,567 families
	Dollars		*Dollars*		*Dollars*
1890....	318.20	1896....	296.76	1902....	344.61
1891....	322.55	1897....	299.24	1903....	342.75
1892....	316.65	1898....	306 70	1904....	347.10
1893....	324.41	1899....	309.19	1905....	349.27
1894....	309.81	1900....	314.16	1906....	359.53
1895....	303.91	1901....	326.90		

33. Relative wholesale prices of raw and manufactured commodities in the United States, 1890–1906:

YEAR	Raw Commodities	Manufactured Commodities	YEAR	Raw Commodities	Manufactured Commodities
1890....	115.0	112.3	1899....	105.9	100.7
1891....	116.3	110.6	1900...	111.9	110.2
1892....	107.9	105.6	1901....	111.4	107.8
1893....	104.4	105.9	1902....	122.4	110.6
1894....	93.2	96.8	1903....	122.7	111.5
1895....	91.7	94.0	1904....	119.7	111.3
1896....	84.0	91.9	1905....	121.2	114.6
1897....	87.6	90.1	1906....	125.9	121.6
1898....	94.0	93.3			

34. Amount of money in circulation per capita in the United States, 1884–1907:

YEAR	Money in circulation per capita	YEAR	Money in circulation per capita	YEAR	Money in circulation per capita
	Dollars		*Dollars*		*Dollars*
1884....	22.65	1892....	24.56	1900....	26.94
1885....	23.02	1893....	24.03	1901....	27.98
1886....	21.82	1894....	24.52	1902....	28.43
1887....	22.45	1895....	23.20	1903....	29.42
1888....	22.88	1896....	21.41	1904....	30.77
1889....	22.52	1897....	22.87	1905....	31.08
1890....	22.82	1898....	25.15	1906....	32.32
1891....	23.42	1899....	25.58	1907....	32.22

35. Receipts and expenditures per capita in the United States:

YEAR	Receipts	Expenditures	YEAR	Receipts	Expenditures
1898....	$6.77	$7.29	1993....	$8.59	$7.920
1899.. .	8.21	9.41	1904....	8.36	8.868
1900....	8.78	7.73	1905....	8.37	8.649
1901....	8.99	7.994	1906....	9.01	8.702
1902....	8.65	7.496	1907....	9.84	8.859

36. Debt per capita less cash in the Treasury of the United States:

YEAR	Debt per cap. less cash in Treas.	YEAR	Debt per cap. less cash in Treas.	YEAR	Debt per cap. less cash in Treas.
	Dollars		*Dollars*		*Dollars*
1881....	35.46	1890....	14.22	1899....	15.55
1882....	31.91	1891. ..	13.34	1900....	14.52
1883....	28.66	1892....	12.93	1901....	13.45
1884....	26.20	1893....	12.64	1902....	12.27
1885....	24.50	1894....	13.30	1903....	11.51
1886....	22.34	1895....	13.08	1904....	11.83
1887....	20.03	1896....	13.60	1905....	11.91
1888....	17.72	1897....	13.78	1906....	11.46
1889....	15.92	1898....	14.08	1907....	10.22

37. Tables showing progress of the United States:

YEAR	Population per sq. mile	Wealth per capita (in dollars)	Cash in Treasury per capita (in dollars)	Circulation per capita (in dollars)	Import of M'd'se per capita (in dollars)	Export of M'd'se per capita (in dollars)
1800.....	6.41	......	15.63	5.00	17 19	13.37
1810.....	3.62	......	7.34	7.59	11.80	9.22
1820.....	4.68	......	9.42	6.94	7.72	7.22
1830.....	6.25	......	3.77	6.79	4.87	5.57
1840.....	8.29	......	.21	10.91	5.76	7.25
1850.....	7.78	307.69	2.74	12.02	7.48	6.23
1855.....	9.14	......	1.31	15.34	9.46	8.03
1860.....	10.39	513.93	1.91	13.85	11.25	10.61
1865.....	11.48	......	76.98	20.57	6.87	4.78
1870. ...	12.74	779.83	60.46	17.50	11.06	9.77
1875.....	14.51	......	47.53	17.16	11.97	11.36
1880....	16.57	850.20	38.27	19.41	12.51	16.43
1885.....	18.55	......	24.50	23.02	10.32	12.94
1890.....	20.69	1,038.57	14.22	22.82	12.35	13.50
1895.....	22.77	1,117.01	13.08	23.20	10.61	11.51
1900.....	25.14	1,164.79	14.52	26.94	10.88	17.96
1905.....	27.38	......	11.91	31.08	13.08	17.94
1906.....	27.82	......	11.45	32.32	14.41	20.40
1907.....	28.35	......	10.22	32.22	16.55	21.60

38. Number of newspapers and periodicals published in the United States:

YEAR	NUMBER	YEAR	NUMBER	YEAR	NUMBER	YEAR	NUMBER
1800..	....	1840..	....	1875..	7,870	1995..	20,395
1810..	359	1850..	2,526	1880..	9,723	1900..	20,806
1820..	861	1860..	4,051	1885..	13,494	1905..	23,146
1830..	1,403	1870..	5,781	1890..	16,948	1907..	21,735

39. The number of students in colleges, universities, and schools of technology in the United States:

YEAR	NUMBER	YEAR	NUMBER
1875........	32,175	1896........	86,864
1880........	38,227	1900........	98,923
1885........	42,573	1903........	108,381
1891........	58,405		

40. The number of volumes in all libraries in the United States:

YEAR	NUMBER (per 100 inhabitants)	YEAR	NUMBER (per 100 inhabitants)
1875........	26	1896........	47
1885........	35	1900........	59
1891........	41	1903........	68

41. According to the "Revista Scientifico-Industriale" the cost of sugar at London and Paris from the middle of the 13th century was as follows:

YEAR	LONDON	PARIS	YEAR	LONDON	PARIS
1260....	$1 87	....	1542....	....	$ .62
1300....	2.27	....	1550....	$ .83	....
1350....	1.51	....	1598....	....	.97
1372....	....	$5.17	1600....	.72	....
1400....	2.10	....	1650....	.73	....
1426....	....	2.62	1700....	.48	....
1450....	2.72	....	1750....	.19	....
1482....	....	2.50	1800....	.34	....
1500....	.48	....			

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